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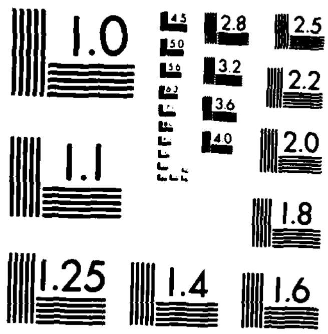

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INSTABILITY AT THE INTERFACE BETWEEN TWO SHEARING FLUIDS IN A CHANNEL

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ABSTRACT

We consider the linear stability of plane Couette flow composed of two immiscible fluids in layers. The fluids have different viscosities but the same densities. It is known that the short wavelength asymptotics of the interfacial mode for the bounded and unbounded problems are identical. In this paper, we show that there is a critical Reynolds number above which the interfacial modes of the unbounded problem are approximated by those of the bounded problem for *wavelengths outside the asymptotic short wavelength range.*

A full linear analysis reveals unstable situations missed out by the long and short wavelength asymptotic analyses but which have comparable orders of magnitudes for the growth rates. The inclusion of a density difference as well as a viscosity difference is discussed. In particular, the arrangement with the heavier fluid on top can be linearly stable in the presence of gravity if the viscosity stratification, volume ratio, surface tension, Reynolds number and Froude number are favorable.

AMS (MOS) Subject Classifications: 76E05, 76T05, 76V05

Key Words: hydrodynamic instability, two-component flows

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SIGNIFICANCE AND EXPLANATION

Shearing flows composed of several superposed liquid layers are frequently encountered in modern coating technology. The stability of a multi-layered flow is especially important in the precision coating of a color film which can consist of many layers. The stability of thin layers discussed in this paper is relevant to flows in the lubrication industry.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

INSTABILITY AT THE INTERFACE BETWEEN TWO SHEARING FLUIDS IN A CHANNEL

Yuriko Renardy

§1. INTRODUCTION

Examples of shearing flows that are composed of a number of different immiscible fluids include the pipeline transport of viscous oils with the addition of water, the production of bicomponent fibers, the coating of color films, and lubrication. Among these examples are situations where the fluids have very different viscosities but similar densities and the speeds are slow. Hence, we focus on the role of viscosity stratification in the linear stability analysis of plane Couette flow of two layers of fluids at low Reynolds numbers.

Two immiscible fluids of given viscosities and densities are confined between infinite parallel plates. The upper plate moves at a given speed in its own plane. In each fluid, the Navier-Stokes equations and incompressibility hold. At the interface, the kinematic free surface condition holds, velocity and shear stress are continuous, and the jump in normal stress is balanced by surface tension. These equations exhibit non-uniqueness since the two fluids can be arranged in any number of horizontal layers with flat interfaces. We choose to consider stability of the arrangement in two layers. Dimensionless parameters are a Reynolds number, a viscosity ratio, a depth ratio and a surface tension parameter. In the presence of a density difference, we have a density ratio and a Froude number. We use a normal mode analysis, in which disturbances are periodic in the horizontal direction and the response depends exponentially on time. The linear stability analysis is set up as an eigenvalue problem, in which the eigenvalue is the coefficient of time in the exponential dependence of the response, and all the other parameters are given.

It is known that plane Couette flow of one fluid is linearly stable at all Reynolds numbers. If a flat horizontal interface is added to the flow with one fluid, the interfacial conditions give rise to an interfacial mode that is neutrally stable, as well as the eigenvalues

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of the one-fluid problem. If the fluids separated by the interface have different mechanical and thermal properties, then the interfacial mode can be unstable even at low Reynolds numbers, and this instability may manifest itself in a wavy interface.

Yih[1] considered the long wavelength asymptotics for two-layered plane Couette and Poiseuille flows for fluids of identical densities. He found that, depending on the viscosity ratio and the volume ratio, the flow can be unstable at any Reynolds number. Yih's asymptotic method has been applied to the linear stability of other multi-component flows, for example, pipe flow with one fluid encapsulating a second fluid[3], layered flow down an inclined plane [4,5], and three-layered Couette and Poiseuille flows[6].

The asymptotic analysis of the interfacial modes for short wavelengths was given by Hooper & Boyd[2]. They analyzed unbounded Couette flow of two fluids. The short wavelength limit of the interfacial modes for this coincides with the short wavelength limit of the interfacial modes of more general parallel shear flows of two fluids because the effect of boundaries turns out to be negligible in that limit. Their asymptotic method has been applied to the flow of two fluids between concentric cylinders with the outer cylinder fixed and the inner cylinder rotating[7]. The asymptotic results have been correlated with numerical studies of the full linear stability problem for the pipe flow[8] and the flow between cylinders[7]. If surface tension is absent, discontinuously viscous stratified flows are unstable to short wavelength disturbances. These instabilities are suppressed by surface tension. This is contrary to expectation because in flows of one fluid, short waves are suppressed by viscosity. Stability in the short-wave limit is governed to a large extent by surface tension, and to a lesser extent by the density and viscosity differences. There has been a conjecture[8] that these short-wave instabilities may be a mechanism for the formation of emulsions. The behavior of small emulsions have often been described by two-phase flow equations that model the emulsion and the surrounding fluid as one material[9]. On the other hand, relatively large emulsions have been observed and sustained by the shearing motion of two different fluids[10]. These appear to have a typical wavelength but we show in §2 that the linear model does not correlate with the experimentally observed length scales[10]. Further work is required to investigate the formation of emulsions, from fingering instabilities at the interface and cusping, which is a three-dimensional nonlinear time-dependent free-surface phenomenon.

There has been much interest in the short wavelength instabilities from both the theoretical and experimental points of view. Hinch[11] has remarked on the practical difficulties of observing the asymptotic short wavelength instabilities by estimating the orders of magnitude which the mechanical properties of the fluids ought to satisfy in order to exhibit such instabilities. In practice, on top of these restrictions, the dimensions of the boundaries is important. We discuss this further in §2. Unbounded Couette flow has only one length scale, namely the length scale of diffusion of momentum, which is derived from the viscosity and velocity gradient in one of the fluids. The presence of boundaries introduces a second length scale, the distance between the boundaries. A Reynolds number can be defined using the scale of boundary separation, velocity scale, and viscosity of one of the fluids. This Reynolds number represents the ratio of squares of the two length scales. As the Reynolds number increases, the range of wavelengths over which the unbounded problem approximates the bounded problem increases from the short wave limit to longer waves.

In §2, we show that a full linear stability calculation reveals a range of instabilities which is in neither the short nor long wavelength limit, but which has comparable growth rates. This band of instabilities occurs for relatively small wavenumbers, but are not long waves because for a fixed Reynolds number, there is stability in the limit of long waves. Hence, the behavior for wavelengths that are neither long nor short cannot simply be interpolated from the asymptotics.

In §3, we include a density stratification and gravity as well as a viscosity stratification. We examine the possibility of a "lubrication stabilization"[7]. This phenomenon occurs in the presence of a small amount of the less viscous fluid and a large amount of the more viscous fluid. There is a tendency for the less viscous fluid to "lubricate" the more viscous fluid and to protect it from shearing. The stability of such a configuration may be strong enough to overcome an adverse density difference. This stabilizing mechanism is absent if the basic flowfield has no shearing[12], such as in rigid-body rotation of two immiscible fluids.

§2. EMULSIONS, SHORT-WAVE INSTABILITIES AND THE PRESENCE OF BOUNDARIES.

Two immiscible fluids of density ρ lie in layers between infinite parallel plates, a distance l^* apart. The upper plate is moving with speed U. Fluids 1 and 2 refer to the lower and upper fluids respectively. The fluids have viscosities μ_i, $i = 1,2$. We define a viscosity ratio $m = \mu_1/\mu_2$. The velocity, distance, time and pressure are made dimensionless with respect to U, l^*, l^*/U and ρU^2 respectively. The Reynolds number in each fluid is $Re_i = Ul^*/\nu_i$ where ν_i is the kinematic viscosity of fluid i. Fig. 1 displays the problem in dimensionless form.

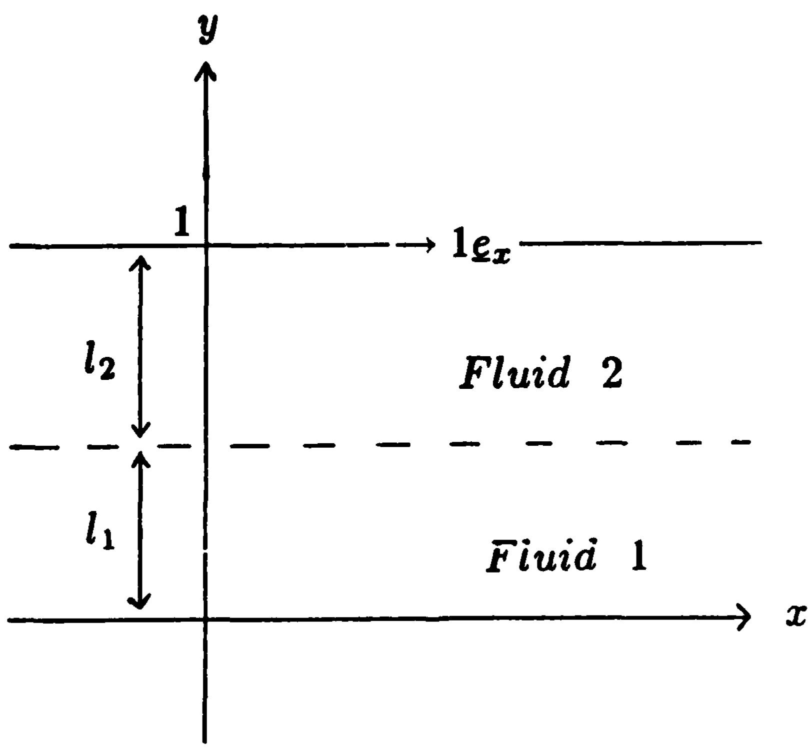

Fig.1

Geometry of problem in dimensionless variables.

In each fluid, the Navier-Stokes equations and incompressibility hold. The boundary conditions are the no-slip conditions. At the interface, the kinematic free-surface condition holds, velocity and shear stress are continuous, and the difference in the normal stress is balanced by surface tension. These equations are satisfied by a flat interface at $y = l_1$ and a dimensionless velocity $(U_1(y),\ 0)$ given by

$$U_1(y) = \frac{y}{l_1 + ml_2} \quad \text{for} \quad 0 \leq y \leq l_1 \tag{1}$$
$$= \frac{m(y-1)}{l_1 + ml_2} + 1 \quad \text{for} \quad l_1 \leq y \leq 1.$$

We consider the linear stability of this basic flowfield by superposing a small disturbance which is proportional to $exp(i\alpha x + \sigma t)$, to the velocity and interface position.

We denote the perturbations to the velocity and interface position by (u, v) and h respectively. We refer to Yih (1967) for the derivation of the following equations. We obtain, in each fluid,

$$Re_i(\sigma + i\alpha U_1(y))(v_{yy} - \alpha^2 v) = \nabla^2(v_{yy} - \alpha^2 v). \tag{2}$$

The interface conditions, linearized at $y = l_1$, yield:

$$v = h(\sigma + i\alpha U_1(l_1)), \quad [\![v]\!] = 0, \quad i[\![v_y]\!] + \frac{\alpha(1-m)h}{l_1 + ml_2} = 0,$$
$$mv_{1yy} - v_{2yy} + \alpha^2 v_1(m-1) = 0, \tag{3}$$

$$\alpha Re_2[\![v_y]\!] = -Th\alpha^4 + mv_{1yyy} - v_{2yyy} - m(3\alpha^2 + Re_1 i\alpha U_1(l_1))v_{1y}$$
$$+(3\alpha^2 + Re_2 i\alpha U_1(l_1))v_{2y} + Re_2 i\alpha v_1 \tfrac{(1-m)}{(l_1+ml_2)},$$

where subscript i refers to Fluid i, our surface tension parameter T is (the surface tension coefficient)/$(\mu_2 U)$, and $[\![\circ]\!]$ denotes $\circ_1 - \circ_2$. For our numerical results, we discretize the equations in the y-direction by using a spectral method based on an expansion in Chebyschev polynomials (Orszag 1971).

In Joseph, Nguyen & Beavers (1984), Figs. 15(c), 16 and 19 - 23 show that relatively large emulsions can be present in shearing flows of two fluids. These bubbles may be called "dynamic" emulsions since they are dynamically maintained by the shearing and disappear when the apparatus is stopped (private communication from D. D. Joseph). There are also smaller bubbles (Drew 1983) which stay in the flow after the apparatus is stopped. The larger bubbles appear to have a typical wavelength, which tempts one to explain their size by the following simple-minded argument. Suppose we model these flows as locally plane Couette flows. The linear theory can yield an unstable range of medium wavelengths λ, say $\lambda_S \leq \lambda \leq \lambda_L$, depending on the volume ratio, viscosity ratio and surface tension. The volume ratio and viscosity ratio determine the stability of long waves[1]; suppose these parameters are such as to make the waves stable for $\lambda \geq \lambda_L$. Surface tension sets a lower bound λ_S on the unstable wavelengths[2]. Is there any agreement between the wavelengths $\lambda_S \leq \lambda \leq \lambda_L$ and the size of the bubbles? We find that λ_S and λ_L both turn out to be too large compared with bubble diameters, or that the linear theory predicts stability. This indicates that the short wavelength instability found by Hooper & Boyd (1983) need not be present to support emulsions. Hence, the dimensions of the boundaries play a crucial role and the above-referenced flows should not be modelled as plane Couette flows. In this context, a more relevant mechanism of instability would be the medium to long wavelength one. For the case of the medium wavelength instability, the Hopf bifurcation (Renardy, M. & Joseph) to a traveling wave may be unstable in some parameter range. For the case of long waves in plane Couette and Poiseuille flows, Hooper & Grimshaw have used the method of multiple scaling to find an amplitude evolution equation for long wavelength weakly nonlinear waves. For instabilities arising from either the medium or long wavelengths, a highly nonlinear wavy interface may result, and the tips of the waves may finger causing bubbles to break off. In this situation, there need be no correlation between the range of unstable wavelengths in the linear stability analysis and the size of the bubbles.

We next examine the relationship between the problem without boundaries and the problem with boundaries. In what follows, the subscript HB denotes the variables for the unbounded problem (see §4 of Hooper & Boyd 1983).

The natural length scale in the unbounded problem is the scale of diffusion of momentum, and for the bounded problem, it is the plate separation. Hence, the dimensionless wavenumber, surface tension parameter and eigenvalue of the two problems are related by

$\alpha_{HB}(\frac{Re_1}{l_1+ml_2})^{1/2} = \alpha, \quad S_{HB}m(Re_1(l_1+ml_2))^{-1/2} = T$ and

$-i\alpha_{HB}C_{HB} = (l_1+ml_2)(\sigma + i\alpha U_1(l_1)).$

In the short wavelength limit, the interfacial eigenvalues of both problems are identical. For the flow with boundaries,

$$\sigma \sim -i\alpha U_1(l_1) + \frac{Re_1 m(1-m)^2}{2(l_1+ml_2)^2\alpha^2(1+m)} - \frac{\alpha T}{2(1+m)} \quad \text{as } \alpha \to \infty, \tag{4}$$

keeping all other parameters fixed. The asymptotic formula (4) is useful if the dimensional wavelength is small compared to both the plate separation ($\alpha >> 1$) and the length scale of diffusion of momentum ($\alpha_{HB} >> 1$). This often requires α to be very large. However, the unbounded problem would approximate the bounded problem for α *out* of this asymptotic range if the plate separation is large compared with the length scale of diffusion of momentum, and if the wavelength under consideration is of the order of the latter. In Fluid 1, the length scale of diffusion of momentum is given by a combination $l^*(\frac{l_1+ml_2}{Re_1 m})^{1/2}$ of the velocity gradient $\frac{Um}{(l_1+ml_2)l^*}$ and the kinematic viscosity. Hence, the ratio of this relative to plate separation is $(\frac{l_1+ml_2}{Re_1 m})^{1/2}$, and this must be small. This indicates that there is a critical Reynolds number, above which the unbounded problem approximates the bounded problem for waves that are short (with respect to plate separation) but not short enough (with respect to diffusion of momentum) to be in the asymptotic range of equation (4).

The dependence of marginal stability on the Reynolds number for the bounded problem is illustrated in Fig.2. The viscosity ratio ($m = 0.5$) and volume ratio ($l_1 = 1/3$) were chosen so that long waves would be stable. The suitable range for these parameters are found in Figs.2(a) and 2(b) of Yih (1967). The surface tension parameter was chosen so that we can compare with Fig.3(a) of Hooper & Boyd (1983) for $S_{HB} = 0.1$, from which we read instability for approximately $0.14 < \alpha_{HB} < 1.3$. Physically, Fig.2 can be interpreted as keeping the fluid properties (surface tension coefficient, μ_1, m, ρ), the volume ratio and the velocity gradients fixed, and varying the plate separation. By $Re_1 = 800$, we find a band of instabilities corresponding to the unbounded problem. At this value of Re_1, the value of α at the upper end of the band is not yet in the short-wavelength range described by (4). This would be the only band of instabilities if the qualitative features of the asymptotic and unbounded analyses were simply interpolated to describe the full linear stability analysis. However, there is another band of instabilities. The dimensional wavenumbers of this band tend to 0 as the plate separation increases. This is not a long-wave instability because at each value of the plate separation, there is stability as $\alpha \to 0$. Maximum growth rates in the two bands of instabilities are of the same order.

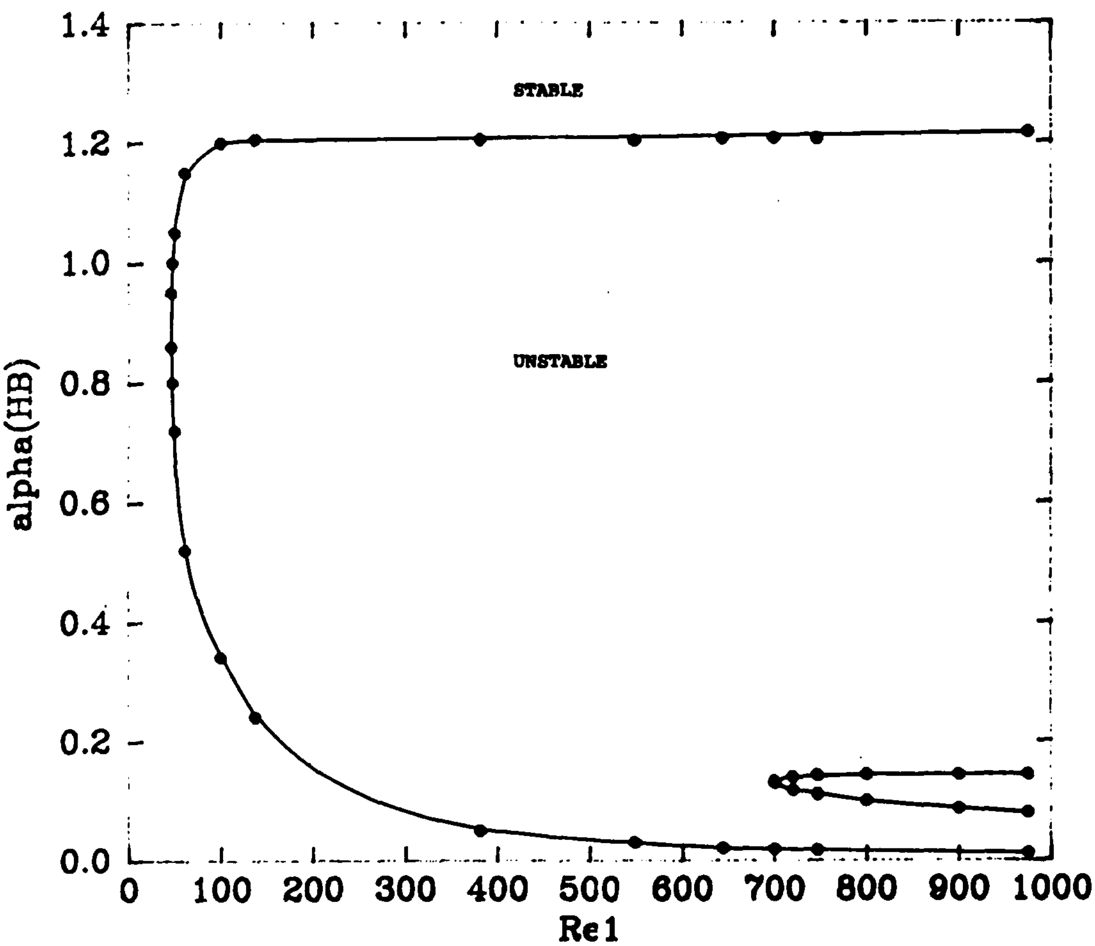

Fig.2

$m = 0.5$, $l_1 = 1/3$, $U = 12$cm/sec, $\mu_1 = 2p$, $\mu_2 = 4p$, $\rho = 1$g/cm/sec,

$S = .75$dyn/cm, $(S_{HB} = 0.1)$, U/l^* fixed at 4.72/sec, $Re_1 = \sqrt{Ul^*/\nu_1}$.

§3. INCLUSION OF DENSITY DIFFERENCE AND GRAVITY

We include a density difference and gravity into the linear stability analysis of §2. We define a Froude number F where $F^2 = U^{*2}/gl^*$, and g is the gravitational acceleration constant. We denote the density ratio ρ_1/ρ_2 by r. The basic velocity profile remains as (1). Gravity introduces a pressure gradient in the basic pres. .re field P given by

$$\frac{\partial P}{\partial y} = -\frac{1}{F^2} \quad \text{for} \quad 0 \leq y \leq l_1,$$

$$= -\frac{1}{rF^2} \quad \text{for} \quad l_1 \leq y \leq 1. \tag{5}$$

This pressure gradient enters into the balance of the normal stress at the interface. The other interfacial conditions remain as in (3). The normal stress balance requires

$$\sigma Re_2(rv_{1y} - v_{2y}) = -Th\alpha^4 + mv_{1yyy} - v_{2yyy} - m(3\alpha^2 + Re_1 i\alpha U_1(l_1))v_{1y}$$

$$+(3\alpha^2 + Re_2 i\alpha U_1(l_1))v_{2y} + Re_2 i\alpha v_1 \frac{(r-m)}{(l_1 + ml_2)} + \alpha^2 h Re_2 \frac{(1-r)}{F^2}. \tag{6}$$

Equation (2) again holds for v. The short wavelength analysis yields (Hooper & Boyd 1983)

$$\sigma + i\alpha U_1(l_1) \sim \frac{m(1-m)(1-m^2/r)Re_1}{2(1+m)^2\alpha^2(l_1+ml_1)^2} - \frac{\alpha T}{2(1+m)} - \frac{m(1-1/r)Re_1}{2(1+m)\alpha F^2} \tag{7}$$

as $\alpha \to \infty$. Surface tension stabilizes short waves. If Fluid 2 is the heavier, then gravity would be expected to destabilize long waves. However, if Fluid 1 is very much less viscous than Fluid 2, and if its depth is small compared to the depth of Fluid 2, then the flow can be linearly stable even when Fluid 2 is the heavier. The stability of such thin layers is relevant to the lubrication industry and to coating technology. A similar situation occurs in the Taylor flow (Renardy, Y. & Joseph 1985), where two fluids lie between two concentric cylinders of infinite extent with the inner cylinder rotating and the outer cylinder at rest. The configuration with a thin layer of the less viscous but heavier fluid lying next to the inner cylinder is found to be linearly stable provided the rotation rate is sufficiently slow.

When the rotation rate is fast, the heavier fluid is thrown out by the centrifugal force but when the speeds are slow, the viscosity stratification and volume ratio dominate over the centrifugal force.

Fig.3 is a graph of the growth rate $re(\sigma)$ against the wavenumber α, $0 \le \alpha \le 1$, for various depth ratios l_1 for plane Couette flow with the heavier fluid on top. We include a sufficient amount of surface tension in order to stabilize the short waves. The Froude number must not be too small or the long waves will be unstable. We choose m small and r close to 1. For $l_1 = 0.05$ and 0.1, we have stability at all values of α, not just those in Fig.3. For $l_1 = 0.2$ and larger, there is instability at low values of α. The flow is stable at sufficiently large α for each l_1, as can be deduced from equation (7). The Reynolds numbers in each fluid are low.

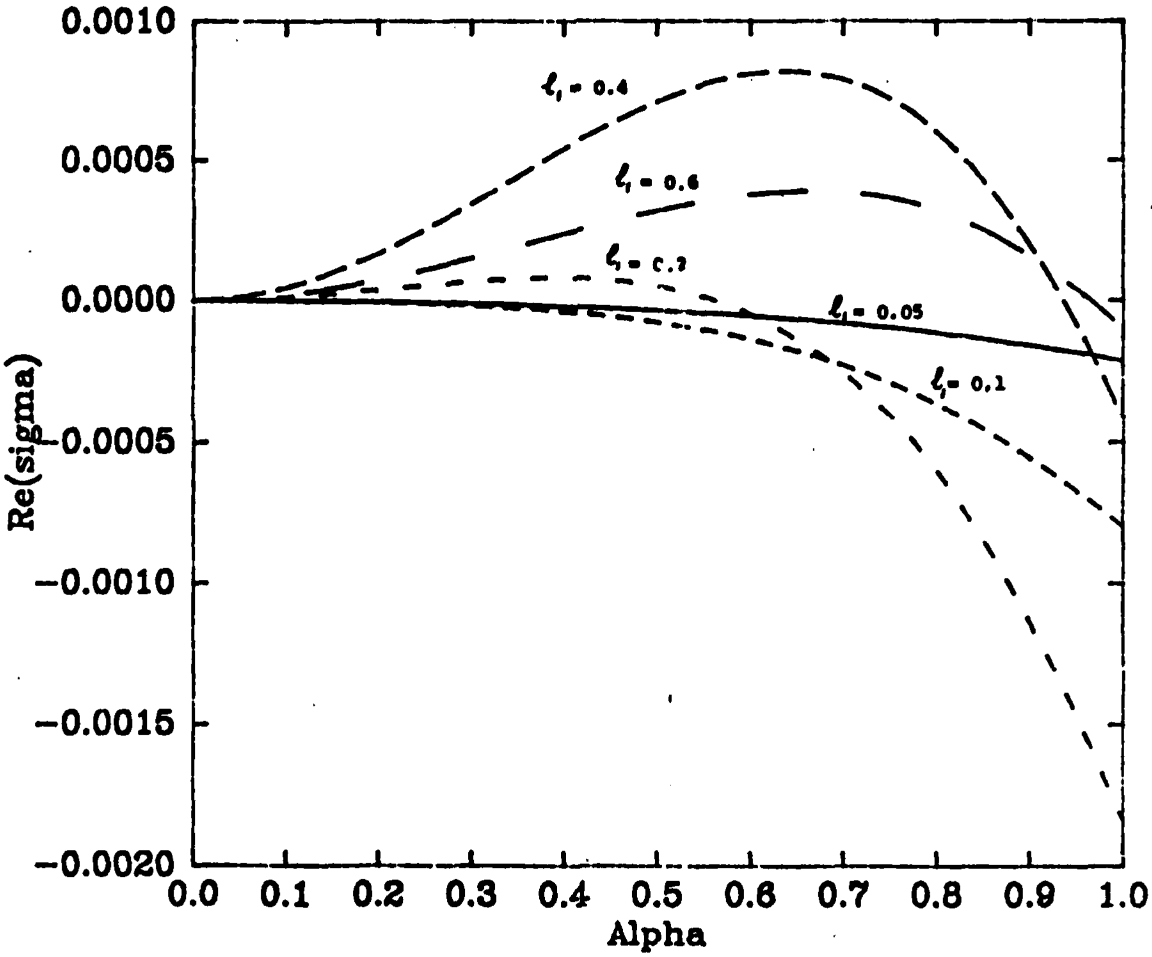

Fig.3

Growth rate vs. α.

$Re_1 = 10$, $m = 0.01$, $r = 0.95$, $T = 0.1$, $\frac{1}{F^2} = 10$.

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